BSc (Maths) part I paper - 2

The straight line (In space)

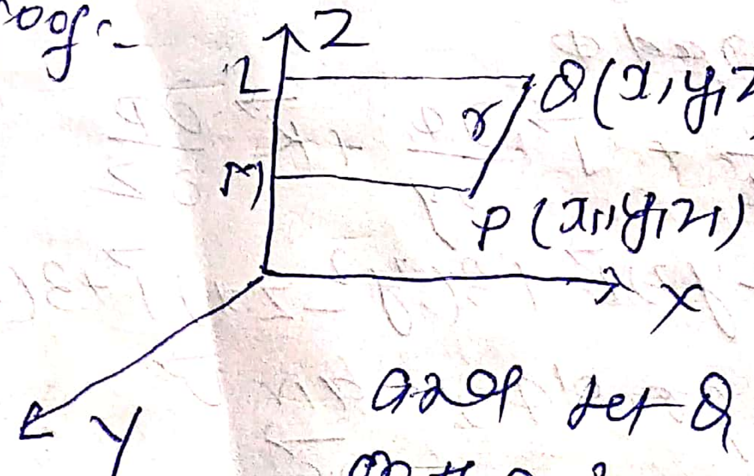
Introduction

A straight line in three dimensional space is determined uniquely by any one of the following geometrical condition:

1. two given point
2. a given point and given direction.
3. intersection of two given different plane.

Theorem ① To find the equation of straight line passing through the given point (x_1, y_1, z_1) whose direction cosine are l, m, n.

proof -



let P be the point whose co-ordinate are (x_1, y_1, z_1) and let Q be other point on the line whose ~~co-ordinates~~ co-ordinate are (x, y, z). Draw PM and QL perpendicular on OZ.

Then $OM = z_1$, $OL = z$, $LM = z - z_1$

Now LM = the projection of PQ on z-axis $= rn$

$\therefore rn = z - z_1 \quad \therefore r = \frac{z - z_1}{n}$

Similarly $\frac{x - x_1}{l} = r$ and $\frac{y - y_1}{m} = r$

Thus the co-ordinate of any point on the line satisfy the relations

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

This is the required eqn. of straight line in symmetrical form.

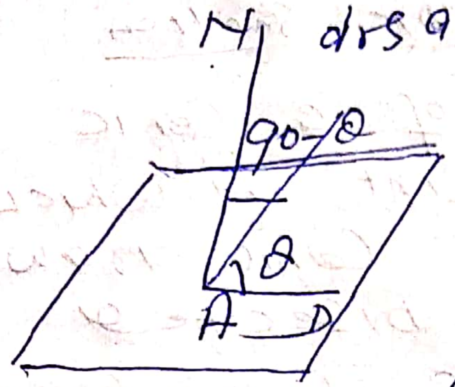
Angle between a line and a plane

Theorem:- To find the angle between the line

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

and the plane $ax + by + cz + d = 0$

proof :-



The equation of the plane is $ax+by+cz+d=0$ the direction ratio of the normal AN to the plane are a, b, c

Then direction cosines are

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Let the eqn. of given line AB be $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$

Let l, m, n be the direction ratio of AB then the direction cosine be

$$\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$$

If θ be the angle between the given line AB and the given plane. then the angle between AB and normal AN will be 90 − θ

Hence $\cos(90-\theta)$

$= l_1 l_2 + m_1 m_2 + n_1 n_2$

(formula)

or $\sin\theta = \dfrac{al + bm + cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$

Distance of a point from line

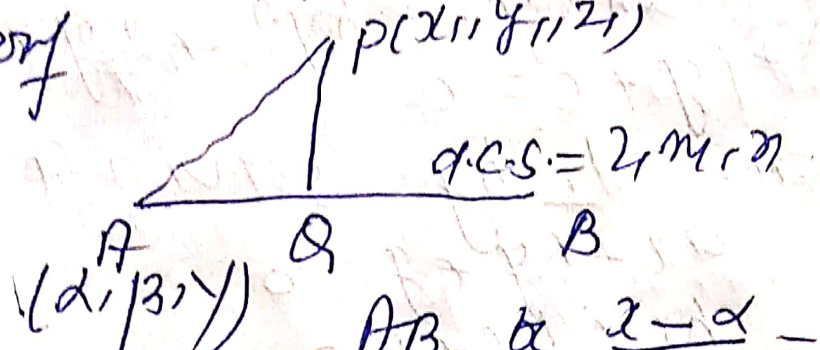
Theorem

To find the perpendicular distance of the point (x_1, y_1, z_1) from the line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

where l, m, n are the direction cosine of the line.

Proof



Let the equations of the line AB is $\dfrac{x-\alpha}{l} = \dfrac{y-\beta}{m} = \dfrac{z-\gamma}{n}$

where l, m, n are the direction cosines of the line and the point A is (α, β, γ)

From the given point $P(x, y, z)$

draw $PQ \perp$ to the given line AB meeting AB at Q. join A and P

then $AP^2 = (x_1-\alpha)^2 + (y_1-\beta)^2 + (z_1-\gamma)^2$

AQ = the lenth of projection of AP on AB.

$= (x_1-\alpha)l + (y_1-\beta)m + (z_1-\gamma)n$

From ~~right~~ right angled tringle AQP, $PQ^2 = AP^2 - AQ^2$

$= (x_1-\alpha)^2 + (y_1-\beta)^2 + (z_1-\gamma)^2$

$- [(x_1-\alpha)l + (y_1-\beta)m + (z_1-\gamma)n]$

this gives the required perpendicular distance PQ

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